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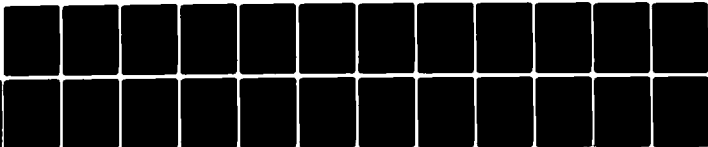
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THE BEHAVIOR OF THE ATMOSPHERE IN THE  
DESERT PLANETARY BOUNDARY LAYER

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low level stratification, and hence the occurrence of such phenomena as radar ducting, dust/sand storms, low level jets.

The general objective of the proposed research is to develop, refine, and intergrate a numerical model of the planetary boundary layer which will have the capability of predicting its future behavior. The model will be tested with reference to an observational mesonet which will be established in the Negev desert.

A feature of the model will be the ability to assess the effect on a mass of warm dry air from the land passing over a body of water. Another related feature of the model will be its ability to predict the variation in desert inversion height - a parameter of utmost importance in controlling desert circulation. The same model will also be applicable in prediction of the behavior of the marine inversion. By including a dust concentration equation the capability of the model will be increased to predict this very important desert parameter.



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MATTHEW J. KERPER  
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## PREFACE

The research described in this report was conducted by personnel of the Desert Meteorology Unit, of The Jacob Blaustein Institute for Desert Research, Ben-Gurion University of the Negev, Sede Boqer, Israel from 1 May 1981 to 30 April 1982 under Grant No. AFOSR 81-0126 to the Ben-Gurion University of the Negev, Research and Development Authority, P.O. B. 1025, Beersheva, Israel.

Participating personnel concerned with the tasks described in this report include Prof. Louis Berkofsky, Principal Investigator, Dr. Avraham Zangvil, Research Associate, Mr. Zvi Shipponey, Mathematician Programmer, and Mr. Eli Zangvil, Programmer.

Observational data utilized in this study were obtained from the Institute's micrometeorological (4m) tower, collected on a 36-channel data logger, and analyzed in the laboratory.

The Director of the Institute during the conduct of this study was Prof. Amos Richmond.

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## Introduction

There exist a large number of planetary boundary layer models each designed for specific purposes, (Deardorff, 1974, Mahrt and Lenschow, 1976, Stull, 1976, Heidt, 1977, Yamada, 1979). Many of these consider the top of the boundary layer a material surface. Some consider the top of the boundary layer to be coincident with the inversion, and consider entrainment across its interface. Some are multi-level, some are bulk, single-level models. The various models are of one, two, and three dimensions. The greater the number of dimensions, the greater the computational complexity.

It is possible to reduce the computational complexity, and still not eliminate the three-dimensionality completely, by using a variation of the "momentum integral" method (Schlichting, 1968). By means of this approach, the vertical structure of several of the variables is specified, and incorporated into the vertically-integrated equations. In this way, the model becomes two-dimensional in the horizontal and the vertical variations are incorporated in various coefficients.

We consider a model of the planetary boundary layer (depth approximately 1 km). This layer is itself divided into a surface layer (approximately 20 m) and a transition layer which at certain times and places becomes very well mixed, and is frequently called the "mixed" layer (Figure 1). We shall not insist that this condition be satisfied everywhere in the model.

Very often, the top of the planetary boundary layer is capped by an inversion. When this is so, and when convection occurs below the inversion, the inversion changes height due to upward and corresponding downward fluxes through it by turbulence. These processes have to be modelled. Further, the processes which we wish to model are on such a scale that fairly high resolution is needed--on the order of 10-20 km in the horizontal, over a region approximately 400-500 km on a side. If then the vertical resolution is to be very fine--say 100 m (above the surface layer), the computation time for solution of only the boundary layer mesoscale equations may become prohibitive. Nevertheless, the optimum mesoscale model must be three-dimensional. As stated above, it may be possible to reduce the computation time considerably by means of a two-dimensional model, and still gain valuable insights by carrying out selected numerical experiments. The results of such experiments will provide information for proper formulation of future three-dimensional mesoscale models. For this reason, we propose to

parameterize quantities in the vertical by assuming that we are permitted to specify their variation with height. If we then integrate the equations vertically up to the inversion height, and introduce the parameterized variables, the vertical variation goes into the coefficients, while the equations themselves apply to parameters at one or two specific levels. The procedure will be described in greater detail below.

#### Model Equations

We shall concern ourselves with a form of the primitive equations derived by averaging over a horizontal area  $\Delta x \Delta y$  which is large enough to contain the sub-grid scale phenomena, but small enough to be a fraction of the mesoscale system. We define

$$(\overline{\quad}) = \frac{1}{\Delta x \Delta y} \iint (\quad) dx dy \quad (1)$$

$$\varphi = \overline{\varphi} + \varphi' \quad (2)$$

where  $\varphi$  is any scalar variable.

With the above definitions, the appropriate equations are, approximately

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}(u^2) + \frac{\partial}{\partial y}(uv) + \frac{\partial}{\partial z}(uw) - f(v - v_g) = -\frac{\partial}{\partial z}(\overline{u'w'}) \quad (3)$$

$$\frac{\partial v}{\partial t} + \frac{\partial}{\partial x}(uv) + \frac{\partial}{\partial y}(v^2) + \frac{\partial}{\partial z}(vw) + f(u - u_g) = -\frac{\partial}{\partial z}(\overline{v'w'}) \quad (4)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (5)$$

$$\frac{\partial H}{\partial t} + \frac{\partial}{\partial x}(uH) + \frac{\partial}{\partial y}(vH) + \frac{\partial}{\partial z}(wH) = \frac{Q}{\rho} - \frac{\partial}{\partial z}(\overline{w'H'}) \quad (6)$$

$$\frac{\partial q}{\partial t} + \frac{\partial}{\partial x}(uq) + \frac{\partial}{\partial y}(vq) + \frac{\partial}{\partial z}(wq) = - \frac{\partial}{\partial z}(\overline{w'q'}) \quad (7)$$

In the above set of equations, the unbarred variables  $u, v, w, u_g, v_g, H, B, q$  are all mean values according to Equation (1). We have used the Boussinesq Assumption, and the variables are defined as follows:

$u, v, w$  = components of the wind vector

$u_g, v_g$  = geostrophic wind components

$H = C_p T + Lq + gz$  = moist static energy

$q$  = specific humidity

$Q/\rho$  = divergence of net radiation flux =  $\frac{\partial F}{\partial z}$

$x, y, z, t$  = space variables and time

Equations (3) and (4) are the horizontal equations of motion, Equation (5) is the continuity equation, Equation (6) is the thermodynamic energy equation, Equation (7) is the specific humidity equation. The lower boundary condition is

$$w = w_T = \nabla \cdot \nabla z_T \text{ at } z = z_T = \text{terrain height} \quad (8)$$

= given

In the above system, we have chosen to treat the moist static energy. The solution for this quantity, together with that for the specific humidity, will enable us to recover the temperature when needed. In the present investigation, we assume that any condensed moisture stays in the air. Thus, we do not treat clouds or precipitation explicitly in this model (but implicit predictions are possible). We indicated earlier that changes of inversion height will have to be modelled. This is particularly important in desertification studies, because many desert areas are located near the descending branch of the Hadley cell, where inversions exist. This can be seen from Israeli data (Shaia and Jaffe, 1976), where there are 222 days, on the average, per year of mid-day

inversions just about 100 km north of the beginning of the Negev desert.

In order to expedite deriving appropriate expressions for predicting inversion height, we first derive inversion "interface" conditions. The results are essentially the upper boundary conditions. Let  $h(x,y,t)$  be the inversion height. Let  $\delta$  be a small layer of constant thickness above the inversion level. In Mahrt and Lenschow (1976) this is called the turbulent inversion or transition layer, which is sufficiently thin so that terms of  $O(\delta)$  may be neglected in the integrated equations.

Define 
$$(\sim) = \frac{1}{\delta} \int_z^{z+\delta} ( ) dz \quad (9)$$

and assume that  $w$  may be replaced by its value  $w_z$  at the bottom of layer.

Let 
$$\Delta ( ) = ( )_{z+\delta} - ( )_z \quad (10)$$

By Leibniz's rule,

$$\int_z^{z+\delta} \frac{\partial}{\partial x_i} ( ) dz = \delta \frac{\partial}{\partial x_i} (\sim) - \Delta ( ) \frac{\partial h}{\partial x_i} \quad (11)$$

where  $x_i = (x, y, t)$

We apply Equation (9) to Equations(3) - (6) inclusive, and obtain

$$\left( \frac{\partial h}{\partial t} - w_z \right) \Delta u + \Delta(u^2) \frac{\partial h}{\partial x} + \Delta(uv) \frac{\partial h}{\partial y} = -(\overline{u'w'})_z \quad (12)$$

$$\left( \frac{\partial h}{\partial t} - w_z \right) \Delta v + \Delta(uv) \frac{\partial h}{\partial x} + \Delta(v^2) \frac{\partial h}{\partial y} = -(\overline{v'w'})_z \quad (13)$$

$$\left(\frac{\partial h}{\partial t} - w_h\right) \Delta H + \Delta F + \Delta(uH) \frac{\partial h}{\partial x} + \Delta(vH) \frac{\partial h}{\partial y} = -(\overline{H'w'})_h \quad (14)$$

$$\left(\frac{\partial h}{\partial t} - w_h\right) \Delta q + \Delta(uq) \frac{\partial h}{\partial x} + \Delta(vq) \frac{\partial h}{\partial y} = -(\overline{q'w'})_h \quad (15)$$

Each of the above equations can be viewed as parameterizations of the vertical eddy fluxes, or as prediction equations for  $h$  if these eddy fluxes are known. The quantity  $\left(\frac{\partial h}{\partial t} - w_h\right)$ , which represents the motion of the air relative to the inversion, is called the "entrainment velocity,"  $w_e$ .  $w_h$  is the larger-scale vertical velocity at the interface. The terms involving  $\frac{\partial h}{\partial x}$  and  $\frac{\partial h}{\partial y}$  are usually omitted in derivations of these interface conditions, since most inversion height models assume horizontal homogeneity.

Our approach will be to integrate the system of Equations (3) - (7) inclusive, with a modified treatment of Equation (5), with respect to  $z$  from  $z = k = (\text{constant}) = \text{height of surface layer}$ , to  $z = h$  (inversion height), and then to introduce modelling assumptions for the variables, i.e., to specify their variations with height. If we do this, it becomes possible to express all of the jump quantities in Equations (12) - (15) inclusive in terms of their values at a specific level.

#### Parameterizations

For the winds in the surface layer, we assume a constant flux profile

$$u(z) = \frac{u_*}{k_1} \ln \left( \frac{z + z_0}{z_0} \right) \quad (16)$$

where  $k_1 = \text{von Karman constant} = 0.4$ ,  $u_* = \text{friction velocity}$ ,  $z_0 = \text{roughness parameter}$ .

In the transition layer, we assume

$$\left. \begin{aligned} u(x, y, z, t) &= A(z) \hat{u}(x, y, t) \\ v(x, y, z, t) &= B(z) \hat{v}(x, y, t) \end{aligned} \right\} \quad (17)$$

where

$$\overline{(\quad)} = \frac{1}{L-k} \int_k^L (\quad) dz \quad (18)$$

We derive a modelling approximation for temperature over land above which a nighttime inversion exists, and where surface heating occurs during the day, leading to convection and turbulent mixing. In Figure 1, an inversion exists in the early morning. The temperature at  $z = k$  is  $T_{ELI}$  where L means land and I means initial value. The temperature increases linearly with height according to

$$T(x, y, z, 0) = T_{RLI}(x, y, 0) + r_i (z - k) \quad (19)$$

is a constant lapse rate.

As convection sets in and  $h$  increases, the temperature changes with height according to

$$T(x, y, z, t) = T_{RL}(x, y, t) + r(z) [L(x, y, t) - k] \quad (20)$$

where, the lapse rate  $r(z)$  need not be constant.

Thus, the temperature jump is seen to be

$$\Delta T = (T_{RLI} - T_{RL}) + \left[ r_i - r(h) \right] (L - k) + r_i \delta \quad (21)$$

Suppose that air flows from land to water, that the water is cooler than the land, and that the air above the water is stably stratified. As the warm air moves over the water, it is cooled from below, i.e., there is a downward transfer of heat from the warm air to the cooler water. At the same time, turbulent mixing is taking place, tending to destabilize the temperature profile. After the mixing process sets in the temperature changes with height according to

$$T(x, y, z, t) = T_h(x, y, t) + \gamma(z) [h(x, y, t) - h] \quad (22)$$

The inversion strength (temperature jump) over water is

$$\Delta T = (T_{hLI} - T_h) + [\gamma_1 - \gamma(h)] (h - h) + \gamma_1 \delta \quad (23)$$

Similarly, the mixing ratio profiles are given by

$$q_f(x, y, z, t) = q_{fLI}(x, y, t) + \delta_1 (z - h) \quad (24)$$

where  $\delta_1$  is constant = initial lapse rate of moisture

$$q_f(x, y, z, t) = q_{fL}(x, y, t) + \delta(z) (h - h) \quad (25)$$

where  $\delta(z)$  = lapse rate of moisture over land

$$q_f(x, y, z, t) = q_{fL}(x, y, t) + \beta(z) (h - h) \quad (26)$$

where  $\beta(z)$  = lapse rate of moisture over water.

The moisture jump is then

$$\Delta q = (q_{fLI} - q_{fL}) + \left[ \delta_1 - \frac{\delta(h)}{\beta(h)} \right] (h - h) + \delta_1 \delta \quad (27)$$

Then the moist static energy is

$$H = H_k + \left\{ c_p \left[ \frac{r(z)}{q(z)} \right] + L \left[ \frac{\delta(z)}{\beta(z)} \right] \right\} (z - k) \quad (28)$$

and the moist static energy jump is

$$\Delta H = (H_{kI} - H_k) + \left\{ c_p \left[ r_1 - \frac{r(k)}{q(k)} \right] + L \left[ \delta_1 - \frac{\delta(k)}{\beta(k)} \right] \right\} (k - k) + (c_p r_1 + L \delta_1) \delta \quad (29)$$

We shall apply the operator Equation (18) to the basic Equations (3) - (7) inclusive, introduce the modelling assumptions Equations (17), (19) - (24) inclusive, and use the interface conditions Equations (12) - (15). However, these will only be used to eliminate  $\overline{(u'w')}_k$  and  $\overline{(v'w')}_k$  from the first two equations of motion Equations (3) and (4). The first law of thermodynamics becomes very complex if we use that procedure there. Instead, we introduce the closure assumption

$$\overline{(w'H')}_k = -A_1 \overline{(w'H')}_k \quad (30)$$

i.e., the turbulent flux of heat at interface height is a fraction of that at the top of the surface layer, and oppositely directed. This assumption has been widely used in the literature (see Carson (1973)). We shall, in fact use the same closure in the interface condition Equation (14) for  $\Delta H$ . The resulting equation will then become the prediction equation for  $h$ .

For the fluxes at  $z = k$ , we write

$$\left. \begin{aligned} \overline{(u'w')}_k &= -C_d |V_k| u_k \\ \overline{(v'w')}_k &= -C_d |V_k| v_k \end{aligned} \right\} \quad (31)$$

$$\left. \begin{aligned} (\overline{q'w'})_R &= E_R \\ (\overline{w'H'})_R &= \frac{H_R^* + LE_R}{P_R} \end{aligned} \right\} \quad (32)$$

$$H_R^* = P_R C_p C_{H0} [A(k) \hat{u}] (T_{GR} - T_R) \quad (33)$$

$$E_R = P_R C_{H0} [A(k) \hat{u}] \frac{W_{GR}}{W_K} [q_{sat}(T_{GR}) - q_R] \quad (34)$$

In the above,  $C_d$  is a turbulent transfer coefficient for momentum,  $C_{H0}$  is a turbulent transfer coefficient for heat,  $T_{GR}$  is ground temperature,  $W_{GR}$  = ground soil moisture,  $W_K$  = potential saturation value of  $W$ .

We shall also parameterize  $\frac{\hat{Q}}{P_R}$  by

$$\frac{\hat{Q}}{P_R} = \frac{S \downarrow (1 - R_{GR}) - 4 \epsilon_{GR} \sigma T_R^4 \left[ \frac{1 - (a_1 + b_1 e^{\frac{1}{2}})}{4} - 1 + \frac{T_{GR}}{T_R} \right]}{P_R (k - k)} = \frac{R_{NET}}{P_R (k - k)} \quad (35)$$

Here  $S \downarrow$  is incoming solar radiation,  $\epsilon_{GR}$  is ground emissivity,  $a_1$  and  $b_1$  are constants associated with downcoming infrared radiation,  $e$  is saturation vapor pressure.  $R_{GR}$  is ground albedo, defined by

$$\left. \begin{aligned} R_{GR} &= a + b \frac{W_{GR}}{W_K}, \quad b < 0 \\ a, b &\text{ constants} \end{aligned} \right\} \quad (36)$$

Thus  $R_{GR}$  decreases as soil moisture increases.

It is seen from the above that we shall need equations for  $T_{GR}$  and  $W_{GR}$ . These will be adapted from Deardorff (1978).

By applying all of the modelling assumptions, we obtain the following:

### Prognostic Equations

These are written for land, for water, we replace  $\gamma$  by  $\alpha$ ,  $\delta$  by  $\beta$ .

$$\left[ \frac{\partial \hat{u}}{\partial t} + \hat{A}^2 \frac{\partial}{\partial x} (\hat{u}^2) + \hat{A} \hat{B} \frac{\partial}{\partial y} (\hat{u} \hat{v}) - f(\hat{v} - \hat{v}_g) \right] + \frac{[A(k+\delta) \omega_k - A(k) \omega_k] \hat{u}}{(k-k)} \quad (37)$$

$$+ [A^2(k) - A^2(k+\delta)] \hat{u}^2 \frac{\partial}{\partial x} \ln(k-k) + [A(k) B(k) - A(k+\delta) B(k+\delta)] \hat{u} \hat{v} \frac{\partial}{\partial y} \ln(k-k) =$$

$$= - \frac{C_d |V_k| A(k) \hat{u}}{(k-k)}$$

### First Equation of Motion

$$\frac{\partial \hat{v}}{\partial t} + \hat{A} \hat{B} \frac{\partial}{\partial x} (\hat{u} \hat{v}) + \hat{B}^2 \frac{\partial}{\partial y} (\hat{v}^2) + f(\hat{u} - \hat{u}_g) + \frac{[B(k+\delta) \omega_k - B(k) \omega_k] \hat{v}}{(k-k)} \quad (38)$$

$$+ [A(k) B(k) - A(k+\delta) B(k+\delta)] \hat{u} \hat{v} \frac{\partial}{\partial x} \ln(k-k) + [B^2(k) - B^2(k+\delta)] \hat{v}^2 \frac{\partial}{\partial y} \ln(k-k) =$$

$$= - C_d \frac{|V_k| B(k) \hat{v}}{(k-k)}$$

### Second Equation of Motion

$$\frac{\partial H_k}{\partial t} + \frac{\partial}{\partial x} (\hat{u} H_k) + \frac{\partial}{\partial y} (\hat{v} H_k) + [C_p (\hat{A} \hat{r}) + L (\hat{A} \hat{\delta})] \frac{\partial}{\partial x} [\hat{u} (k-k)] + [C_p (\hat{B} \hat{v}) + L (\hat{B} \hat{\delta})] \frac{\partial}{\partial y} [\hat{v} (k-k)] + \omega_k \left\{ \frac{H_k}{(k-k)} + [C_p r(k) + L \delta(k)] \right\} \quad (39)$$

$$- \frac{\omega_k H_k}{(k-k)} + \left\{ C_p [2\hat{r} - r(k)] + L [2\hat{\delta} - \delta(k)] \right\} \frac{\partial H}{\partial t} + \frac{[1-A(k)] (H_k \hat{u})}{(k-k)} \frac{\partial}{\partial x}$$

$$+ \frac{[1-B(k)] (H_k \hat{v})}{(k-k)} \frac{\partial}{\partial y} = \frac{\hat{Q}}{P_k} + \frac{(1+A_1) (H_k + L E_k)}{P_k (k-k)}$$

### First Law of Thermodynamics

$$\begin{aligned}
& \frac{\partial q_k}{\partial t} + \frac{\partial}{\partial x} (\bar{u} q_k) + \frac{\partial}{\partial y} (\bar{v} q_k) + (\hat{\delta} - \delta_1) \frac{\partial}{\partial t} (h-k) + \frac{(q_k - q_{kI} - \delta_1 \delta)}{(h-k)} \frac{\partial}{\partial t} (h-k) \\
& + (\hat{\delta} A) \frac{\partial}{\partial x} [\bar{u} (h-k)] + (\hat{\delta} B) \frac{\partial}{\partial y} [\bar{v} (h-k)] + [(\hat{\delta} A) - \delta(h+\delta) A(h+\delta)] \bar{u} \frac{\partial}{\partial x} (h-k) \\
& + [(\hat{\delta} B) - \delta(h+\delta) B(h+\delta)] \bar{v} \frac{\partial}{\partial y} (h-k) + \left[ \frac{1-A(h+\delta)}{(h-k)} \right] (q_k \bar{u}) \frac{\partial}{\partial x} (h-k) \\
& + \left[ \frac{1-B(h+\delta)}{(h-k)} \right] (q_k \bar{v}) \frac{\partial}{\partial y} (h-k) + w_k \left[ \delta_1 - \delta(h) + \frac{(q_{kI} + \delta_1 \delta)}{(h-k)} \right] \\
& - \frac{w_k q_k}{(h-k)} = \frac{E_k}{(h-k)}
\end{aligned} \quad (40)$$

Moisture Equation

$$\begin{aligned}
& [a_1 (h-k) + a_4] \frac{\partial}{\partial t} (h-k) + a_2 \bar{u} \frac{\partial}{\partial x} \frac{(h-k)^2}{2} + a_3 \bar{v} \frac{\partial}{\partial y} \frac{(h-k)^2}{2} \\
& + a_5 \bar{u} \frac{\partial}{\partial x} (h-k) + a_6 \bar{v} \frac{\partial}{\partial y} (h-k) = \\
& = [a_1 (h-k) + a_4] w_k + \frac{A_1 (H_{kI} + L E_k)}{P_k}
\end{aligned} \quad (41)$$

Inversion Height Equation

where

$$\begin{aligned}
a_1 &= c_p [r_1 - r(h)] + L [\delta_1 - \delta(h)] \\
a_2 &= A(h+\delta) (c_p r_1 + L \delta_1) - A(h) [c_p r(h) + L \delta(h)] \\
a_3 &= B(h+\delta) (c_p r_1 + L \delta_1) - B(h) [c_p r(h) + L \delta(h)] \\
a_4 &= [(H_{kI} - H_k) + (c_p r_1 + L \delta_1) \delta] \\
a_5 &= [A(h+\delta) H_{kI} - A(h) H_k] + A(h+\delta) (c_p r_1 + L \delta_1) \delta \\
a_6 &= [B(h+\delta) H_{kI} - B(h) H_k] + B(h+\delta) (c_p r_1 + L \delta_1) \delta
\end{aligned}$$

In the above derivation, we have neglected  $\Delta F$ .

$$\frac{\partial T_{GR}}{\partial t} = - \frac{2\pi^{\frac{1}{2}}}{\rho_s c_s d_1} H_A - \frac{2\pi}{\tau_1} (T_{GR} - T_2) \quad (42)$$

Ground Temperature Equation

where

$$H_A = H_R^* + L E_R - R_{NET}$$

$\rho_s$  = soil density

$T_2$  = deep soil temperature

$C_s$  = soil specific heat

$$d_1 = (h_s \tau_1)^{\frac{1}{2}}$$

$K_s$  = soil thermal diffusivity

$\tau_1$  = period of 1 day

Equation (37) is referred to as the "force restore" equation for ground temperature

$$\frac{\partial W_{GR}}{\partial t} = -C_1 \frac{(E_R - P)}{\rho_w} - C_2 \frac{(W_{GR} - W_2)}{\rho_w \tau_1} \quad (43)$$

#### Soil Moisture Equation

where  $C_1$  and  $C_2$  are constants

$\rho_w$  = density of liquid water

$P$  = precipitation rate (prescribed)

$W_2$  = bulk moisture (analogous to  $T_2$ )

Notice that all capped coefficients, i.e.,  $\hat{A}^2$  etc., are functions of  $h$ .

We still need diagnostic equations.

#### Diagnostic Equations

The parameterizations themselves are diagnostic equations. We have already stated an empirical diagnostic equation for ground albedo (Equation (31)). We still need expressions for the vertical velocity  $w_R$  and the geostrophic wind values  $\hat{u}_g$ ,  $\hat{v}_g$ .

When deriving the appropriate continuity equation from Equation (5) and the lower boundary condition Equation (8), we have to distinguish between two cases - when  $z_T \leq h$ , and  $z_T > h$ . Thus, vertical integration of the continuity equation and the application of Equations (16) and (17) leads to

$$\begin{aligned}
\omega(x, y, z, t) = & -\frac{u_*}{k_1} \ln\left(\frac{z+z_0}{z_0}\right) \frac{\partial z_T}{\partial x} \\
& - \left\{ \left[ \int_k^z A(z) dz \right] \frac{\partial \hat{u}}{\partial x} + \left[ \int_k^z B(z) dz \right] \frac{\partial \hat{v}}{\partial y} \right\}, \quad (44) \\
& z_T \leq k
\end{aligned}$$

$$\begin{aligned}
\omega(x, y, z, t) = & - \left[ A(z_T) \hat{u} \frac{\partial z_T}{\partial x} + B(z_T) \hat{v} \frac{\partial z_T}{\partial y} \right] \\
& - \left\{ \left[ \int_k^z A(z) dz \right] \frac{\partial \hat{u}}{\partial x} + \left[ \int_k^z B(z) dz \right] \frac{\partial \hat{v}}{\partial y} \right\}, \quad (45) \\
& z_T > k
\end{aligned}$$

#### Equations of Continuity

$\omega_k$  can be obtained by setting  $z = h$  in the upper limit of the integrals from the appropriate equation.

In terms of the modelling assumptions, we have

$$\left. \begin{aligned}
\hat{u}_g & \approx - \frac{R}{f} \frac{\partial}{\partial y} \left[ T_{kLI} + \left( \frac{\hat{r}}{q} \right) (h-k) \right] \\
\hat{v}_g & \approx \frac{R}{f} \frac{\partial}{\partial x} \left[ T_{kLI} + \left( \frac{\hat{r}}{q} \right) (h-k) \right]
\end{aligned} \right\} \quad (46)$$

#### Mean Geostrophic Wind Components

The system of prognostic Equations (37) - (43) inclusive, together with the diagnostic Equations (33), (34), (35), (44), (45), (46), comprise the forecast scheme for prognostic variables  $\hat{u}, \hat{v}, H_R, q_R, R, T_{GR}, W_{GR}$  and the diagnostic variables  $w_R, \hat{u}_g, \hat{v}_g, R_{GR}, H_R^*, E_R, \hat{Q}$  provided  $s \downarrow$  and the various functions of  $z$  are known.

It is possible to add a dust concentration equation to this system, for example, a time-dependent equation in which the time rate of change of dust concentration is given by a balance (or imbalance) among sedimentation, diffusion, and transport.

A hierarchy of models based on the above general system can be envisioned.

#### A Hierarchy of Models

The simplest time-dependent model which may be envisioned from the above general model is one in which there is no horizontal advection, and hence no horizontal divergence and no vertical motion. We consider a dry model over flat terrain.

$$\frac{\partial \hat{u}}{\partial t} - f(\hat{v} - \hat{v}_g) + \frac{[1 - A(h+\delta)]}{(h-h)} \hat{u} \frac{\partial}{\partial t} (h-h) = -c_d [A(h)]^2 |\hat{v}| \hat{u} \quad (47)$$

#### 1st Equation of Motion

$$\frac{\partial \hat{v}}{\partial t} + f(\hat{u} - \hat{u}_g) + \frac{[1 - B(h+\delta)]}{(h-h)} \hat{v} \frac{\partial}{\partial t} (h-h) = -c_d [B(h)]^2 |\hat{v}| \hat{v} \quad (48)$$

#### 2nd Equation of Motion

$$\frac{\partial T_R}{\partial t} + [2\hat{r} - r(h)] \frac{\partial}{\partial t} (h-h) = \frac{\hat{Q}}{c_p} + \frac{(1+A_1) c_{H_2O} A(h) \hat{u} (T_{GR} - T_R)}{(h-h)} \quad (49)$$

#### First Law of Thermodynamics

$$\frac{\partial}{\partial t}(h-h) = \left\{ \frac{A_1 C_{H0} A(h) \hat{u} (T_{GR} - T_R)}{[r_1 - r(h)](h-h) + (T_{R1} - T_R)} \right\} \quad (50)$$

### Inversion Height Equation

$$\frac{\partial T_{GR}}{\partial t} = \frac{2\pi^{\frac{1}{2}}}{\rho_s c_s d_1} \left[ R_{WET} - \rho_R c_p C_{H0} A(h) \hat{u} (T_{GR} - T_R) \right] - \frac{2\pi}{\gamma_1} (T_{GR} - T_2) \quad (51)$$

### Ground Temperature Equation

Notice that, if  $h = \text{const.}$ , the two equations of motion reduce to the Ekman equations. According to Equation (46) neglect of horizontal gradient leads to  $u_g = v_g = 0$ . However, we should include a basic current, and this is done by writing

$$\hat{u}_g = \text{constant} \quad (52)$$

The next set in the hierarchy will be derived by adding horizontal transport in the x-direction and vertical motion to the above.

$$\begin{aligned} \frac{\partial \hat{u}}{\partial t} + \hat{A}^2 \frac{\partial \hat{u}^2}{\partial x} - f(\hat{v} - \hat{v}_g) + \left[ \frac{A(h+\delta) \omega_R - A(h) \omega_R}{(h-h)} \right] \hat{u} \\ + \left[ \frac{-A^2(h+\delta)}{(h-h)} \right] \hat{u} \frac{\partial}{\partial t}(h-h) + \left[ \frac{\hat{A}^2 - A^2(h+\delta)}{(h-h)} \right] \hat{u}^2 \frac{\partial}{\partial x}(h-h) = \\ = -C_d [A(h)]^2 |\hat{w}| \hat{u} \end{aligned} \quad (53)$$

### 1st Equation of Motion

$$\begin{aligned}
& \frac{\partial \hat{u}}{\partial t} + \hat{A} \hat{B} \frac{\partial}{\partial x} (\hat{u} \hat{v}) + \hat{f} (\hat{u} - \hat{u}_g) + \left[ \frac{B(h+\delta) w_R - B(h) w_R}{(h-h)} \right] \hat{v} \\
& + \left[ \frac{1-B(h+\delta)}{(h-h)} \right] \hat{v} \frac{\partial}{\partial t} (h-h) + \left[ \frac{A(h) B(h) - A(h+\delta) B(h+\delta)}{(h-h)} \right] \hat{u} \hat{v} \frac{\partial}{\partial x} (h-h) = \\
& = -C_d [B(h)]^2 |\hat{v}| \hat{v}
\end{aligned} \tag{54}$$

### 2nd Equation of Motion

$$\begin{aligned}
& \frac{\partial T_R}{\partial t} + \frac{\partial}{\partial x} (\hat{u} T_R) + (\hat{A} \hat{r}) \frac{\partial}{\partial x} [\hat{u} (h-h)] + \left[ \frac{T_R}{(h-h)} + r(h) \right] w_R \\
& - \frac{T_R w_R}{(h-h)} + [2\hat{r} - r(h)] \frac{\partial}{\partial t} (h-h) + \left[ \frac{1-A(h)}{(h-h)} \right] (T_R \hat{u}) \frac{\partial}{\partial x} (h-h) = \\
& = \frac{\hat{Q}}{C_p} + \frac{(1+A_1) C_{Ho} A(h) \hat{u} (T_{GR} - T_R)}{(h-h)}
\end{aligned} \tag{55}$$

### First Law of Thermodynamics

$$\begin{aligned}
& \frac{\partial}{\partial t} (h-h) + \left\{ \frac{A(h+\delta) r_i - A(h) r(h)}{[r_i - r(h)](h-h) + (T_{RI} - T_R)} \right\} \hat{u} \frac{\partial}{\partial x} \frac{(h-h)^2}{2} \\
& + \left\{ \frac{A(h+\delta) T_{RI} - A(h) T_R}{[r_i - r(h)](h-h) + (T_{RI} - T_R)} \right\} \hat{u} \frac{\partial}{\partial x} (h-h) = w_R \\
& + \left\{ \frac{A_1 C_{Ho} A(h) \hat{u} (T_{GR} - T_R)}{[r_i - r(h)](h-h) + (T_{RI} - T_R)} \right\}
\end{aligned} \tag{56}$$

### Inversion Height Equation

$$\left. \begin{aligned} w_R &= -(h-h) \frac{\partial \hat{u}}{\partial x} \\ w_R &= 0 \end{aligned} \right\} \tag{57}$$

### Equations of Continuity

$$\frac{\partial T_{GR}}{\partial t} = \frac{2\pi^{\frac{1}{2}}}{P_5 S d_1} \left\{ R_{NET} - \rho_A c_p C_{H_2O} A(h) \hat{u} (T_{GR} - T_A) \right\} - \frac{2\pi}{\tau_1} (T_{GR} - T_2) \quad (58)$$

### Ground Temperature Equation

According to Equation (46),  $\hat{u}_g = 0$  unless we have a constant basic flow, i.e.,

$$\left. \begin{aligned} \hat{u}_g &\approx -\frac{R}{f} \frac{\partial}{\partial y} T_{kLT} = \text{constant} \\ \text{also} \quad \hat{v}_g &\approx \frac{R}{f} \frac{\partial}{\partial x} [T_{kLT} + \hat{r}(h-k)] \end{aligned} \right\} \quad (59)$$

Equations (53) - (59) inclusive solve the system for  $\hat{u}$ ,  $\hat{v}$ ,  $T_k$ ,  $h$ ,  $T_{GR}$ ,  $w_k$ ,  $\hat{u}_g$ ,  $\hat{v}_g$ .  $\hat{A}$ ,  $A(h)$ ,  $A(h+\delta)$ ,  $B(h)$ ,  $B(h+\delta)$ ,  $r(h)$  are all functions of  $h$ ;  $k$ ,  $f$ ,  $\hat{u}_g$ ,  $C_d$ ,  $A_1$ ,  $A(h)$ ,  $B(h)$ ,  $R_{GR}$ ,  $C_{H_2O}$ ,  $a_1$ ,  $b_1$ ,  $e^{\frac{1}{2}}$ ,  $c_p$ ,  $\rho_A$ ,  $\hat{r}$ ,  $\tau_1$ ,  $\beta$ ,  $S$ ,  $d_1$ ,  $\tau_1$ ,  $T_2$  are all constants.  $S \downarrow$  is given. The functions  $A(z)$ ,  $B(z)$ ,  $r(z)$  are empirically determined.

These are the next systems in the hierarchy of those we plan to investigate. Following these we will incorporate the moisture in one-dimension, and the terrain in one dimension. We will then go on to two-dimensional (space) models.

### A Test of the System

A very simple test of the system can be made if we consider an even simpler version of the inversion height equation. If, in Equation (56) we omit horizontal advection effects, we obtain

$$\frac{\partial}{\partial t}(h-k) = w_k + \left\{ \frac{[A_1 C_{HO} A(h) \hat{u} (T_{GR} - T_k)]}{[\gamma_1 - \gamma(h)](h-k) + (T_{kI} - T_k)} \right\} \quad (60)$$

This equation is similar to that of Tennekes (1973).

Of course, to test Equation (60), prediction equations are required for  $T_{GR}$ ,  $T_k$ ,  $\hat{u}$ , and a diagnostic equation for  $w_k$ . What we have done is to use real data for the heat flux  $\hat{u}(T_{GR} - T_k)$ , and the temperature anomaly  $(T_{kI} - T_k)$ . For  $w_k$ , we have assumed values shown by the curve in Figure 2. The data are for Sede Boqer, Israel for 18 June 1979. We have assumed an initial value of  $h = 300$  m at 0900 LST. We use  $A_1 = 0.2$ ,  $\gamma_1 = 0.5 \times 10^{-4}$  deg  $cm^{-1}$ ,  $\gamma(h) = -0.5 \times 10^{-4}$  deg  $cm^{-1}$ ,  $C_{HO} = 1.695 \times 10^{-3}$ ,  $k = 20$  m.

The results are shown in Figure 3. On the same figure is the march of heat flux near the ground. Although no verification data are as yet available, it can be seen that the variation of inversion height throughout the day is reasonable. The maximum of 1051 m is reached at 1800. This maximum lags the heat flux by about 6 hours, which is in agreement with the O'Neill, Nebraska results shown in Figure 1 of Carson (1973). Our results are very sensitive to vertical velocity variations. Figure 3 shows the evolution of the inversion height, using the same data as above, but with  $w_h = 0$ , (see Equation (60)). It is clear that the larger-scale vertical velocity plays an important role in determining inversion height, especially at night.

In the near future, we plan to release mini-sondes in order to obtain verification data. We will also attempt to obtain data by means of a tethered balloon. We have an AN/TPQ-11 vertically pointing radar, which is capable of detecting inversions. We hope to have this instrument operable during the coming year.

We have also carried out calculations, based on Equation (60), to test the effect of a water body of the air flowing over it. In this case, we must of course include moisture. Using the various modelling approximations, the inversion height equation becomes (recall that  $w_{GR} = w_k$  over water),

$$\frac{\partial h}{\partial t} = w_R + \frac{A_1 C_{H_2O} A(h) \hat{u} \left\{ (T_{GR} - T_R) + \frac{L}{C_p} [q_{sat}(T_{GR}) - q_R] \right\}}{\left\{ [\gamma_1 - \alpha(h)] + \frac{L}{C_p} [\delta_1 - \beta(h)] \right\} (h - h_R) + [T_{RI} - T_R] + \frac{L}{C_p} (q_{RI} - q_R)} \quad (61)$$

Here, if we wish to feed in data over water, we are in a worse position than previously, as we have no such data for the Sede Boquer area. We must therefore make some heavy assumptions. Suppose we assume that

$$\left\{ (T_{GR} - T_R) + \frac{L}{C_p} [q_{sat}(T_{GR}) - q_R] \right\}_{water} = (T_{GR} - T_R)_{land} \quad (62)$$

and

$$\left[ (T_{RI} - T_R) + \frac{L}{C_p} (q_{RI} - q_R) \right]_{water} = (T_{RI} - T_R)_{land} \quad (63)$$

This is based on the fact that both  $(T_{GR} - T_k)$  and  $(T_{RI} - T_k)$  are smaller than their values over land. However, the latent heat term increases their values. Then the main difference between land and water lies in the factor  $[\gamma_1 - \alpha(h)] + \frac{L}{C_p} [\delta_1 - \beta(h)] = D$ . We can generate a series of results for various values of

$$D = .2, .3, .4, \dots, 2.0, \dots, 3.0 \times 10^{-4} \text{ deg cm}^{-1}$$

Of course, if we keep  $w_h$  the same as that over land, we are assuming the same kind of convection over water as over land. The calculations can also be carried out for various  $w_h$ .

We have carried out one calculation, with  $w_h$  as given in Figure 2. We have used  $D = 1.2 \times 10^{-4}$ . All other conditions were the same as previously. The resulting curve for  $h$  is shown by the middle line in Figure 3. The entire curve, after the first two hours, lies below that for land. The maximum of 950 m is reached at 1800, as before.

The principal conclusion to be drawn is that the inversion undergoes an evolution over water similar to that over land, but that it reaches a lower maximum than that over land. Furthermore, the minimum, in the early morning hours, was also lower than over land. This result has important consequences for problems like radar ducting, dust spreading, and air pollution.

In order to test this result further, it will be necessary to obtain data from a meso-net, both over land and over water. Then the system of Equations (53) - (59) inclusive can be integrated for a thorough test.

#### Conclusions

We have derived a general system of vertically integrated equations for the desert planetary boundary layer. From this system, a hierarchy of models can be designated. We have tested the simplest possible version in the hierarchy, where real desert data were fed in to the inversion height equation. The evolution of the desert inversion was very realistic, although the results have not yet been verified. We have shown that the vertical velocity of the meso-scale motions at inversion height is an important parameter in determining inversion oscillations. This is particularly true at night, when sinking motion tends to depress the inversion. The effect of vertical motion is clearly brought out when the calculations are carried out for zero vertical motion. Then, the large upward flux of heat raises the inversion during the day, but the downward flux during the night is insufficient to lower the inversion height, which remains constant after reaching a maximum in the late afternoon. Using assumed data for water, the inversion height simulation appears realistic, and more or less parallels that over land. But the maximum height is less than that over land, as is the minimum.

### Recommendations

It is recommended that we continue with the next set in the hierarchy. At the same time, we plan to release mini-sondes, launch a tethered balloon, and put into operation a vertically pointing radar, which is capable of detecting inversions. We also plan to gather data from a meso-net for additional verification.

We are purchasing an instrument for sampling the dust content of the desert atmosphere. This instrument will separate the dust into five sizes. We will be able to obtain the individual and total size concentration. These data will serve as input into a numerical dust model, and will also serve as ground truth information for lidar measurements giving dust distribution as a function of height. Details of the derivation of a dust model equation and some preliminary results are given in a Proposal to the Department of the Air Force (AFSC) to extend subject grant.

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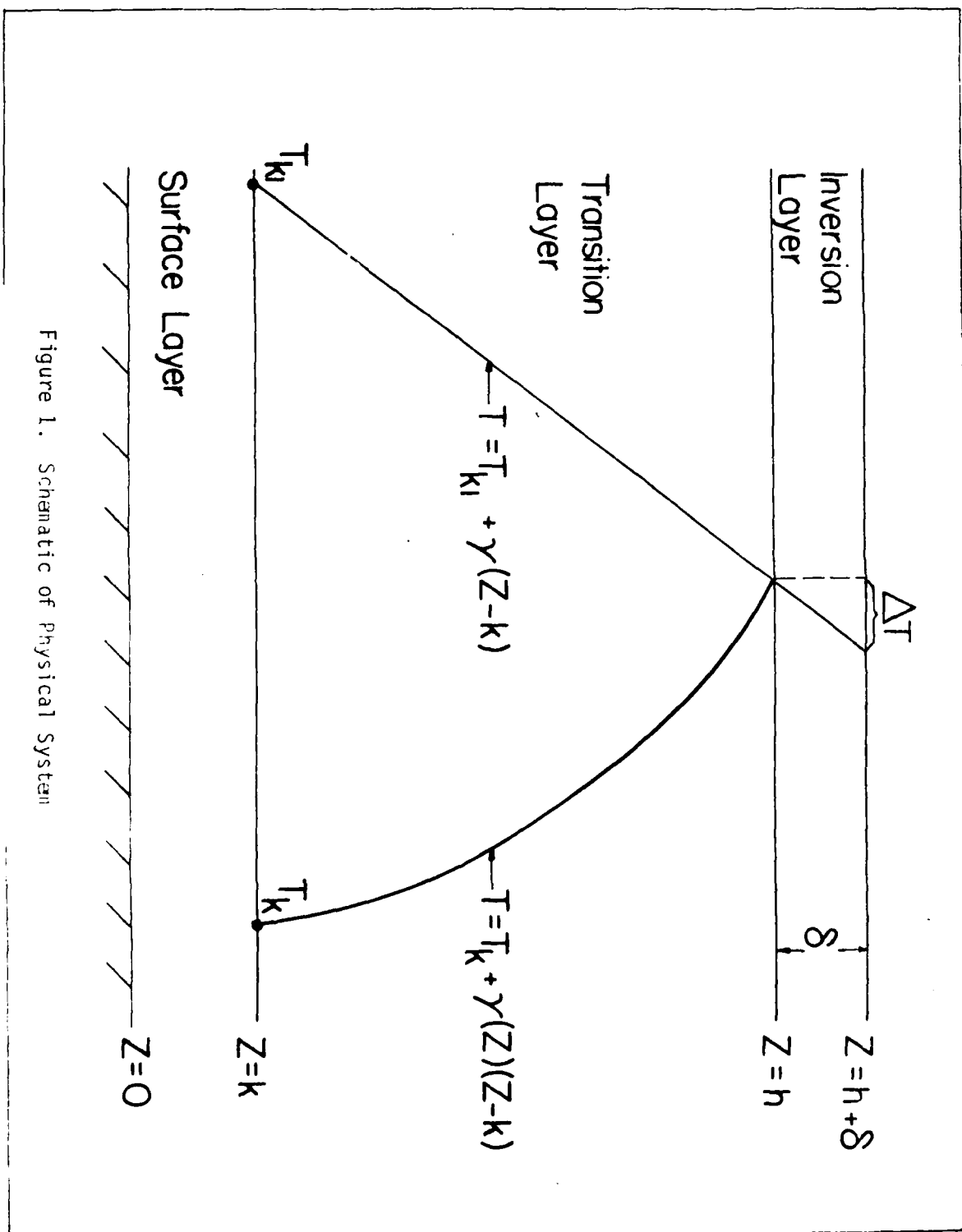


Figure 1. Schematic of Physical System

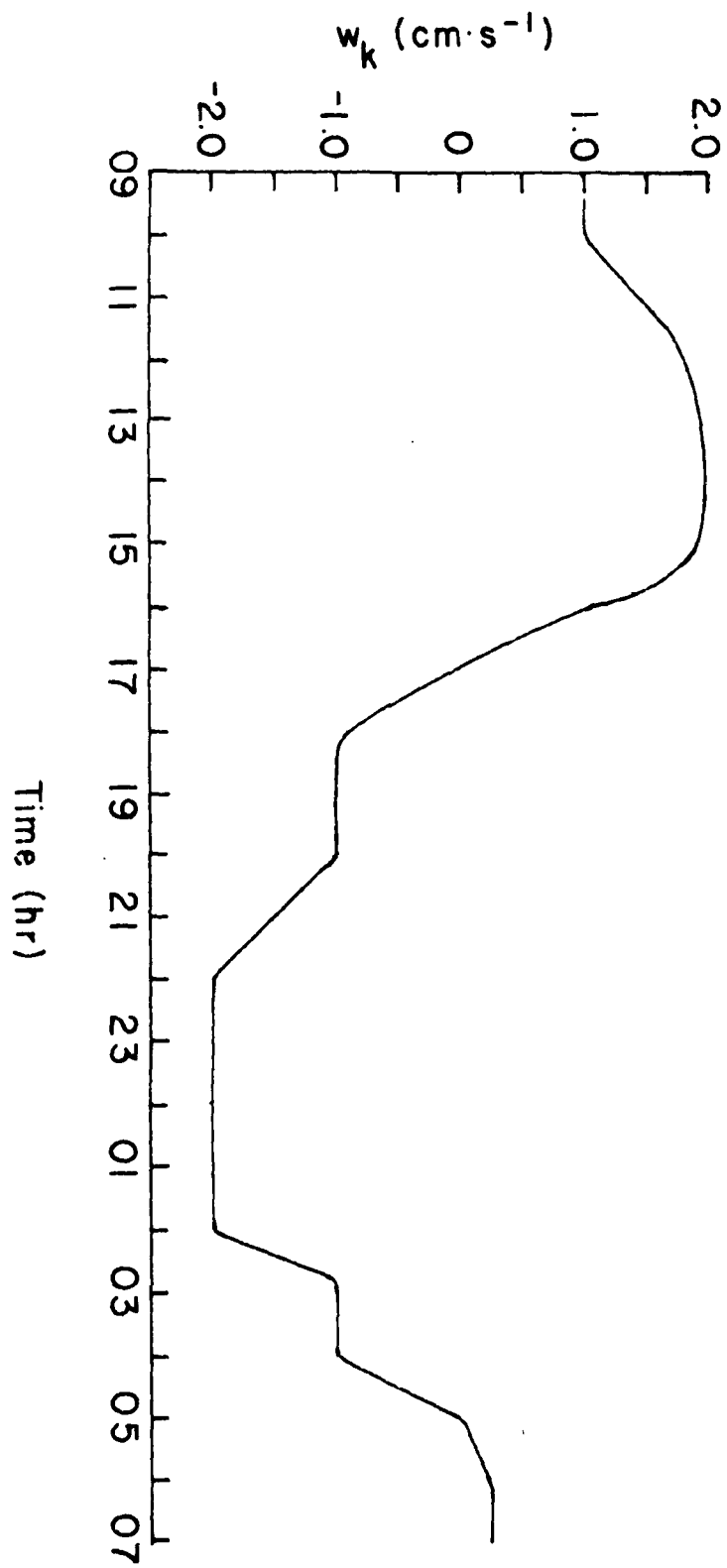


Figure 2. Assumed values of  $w_h$

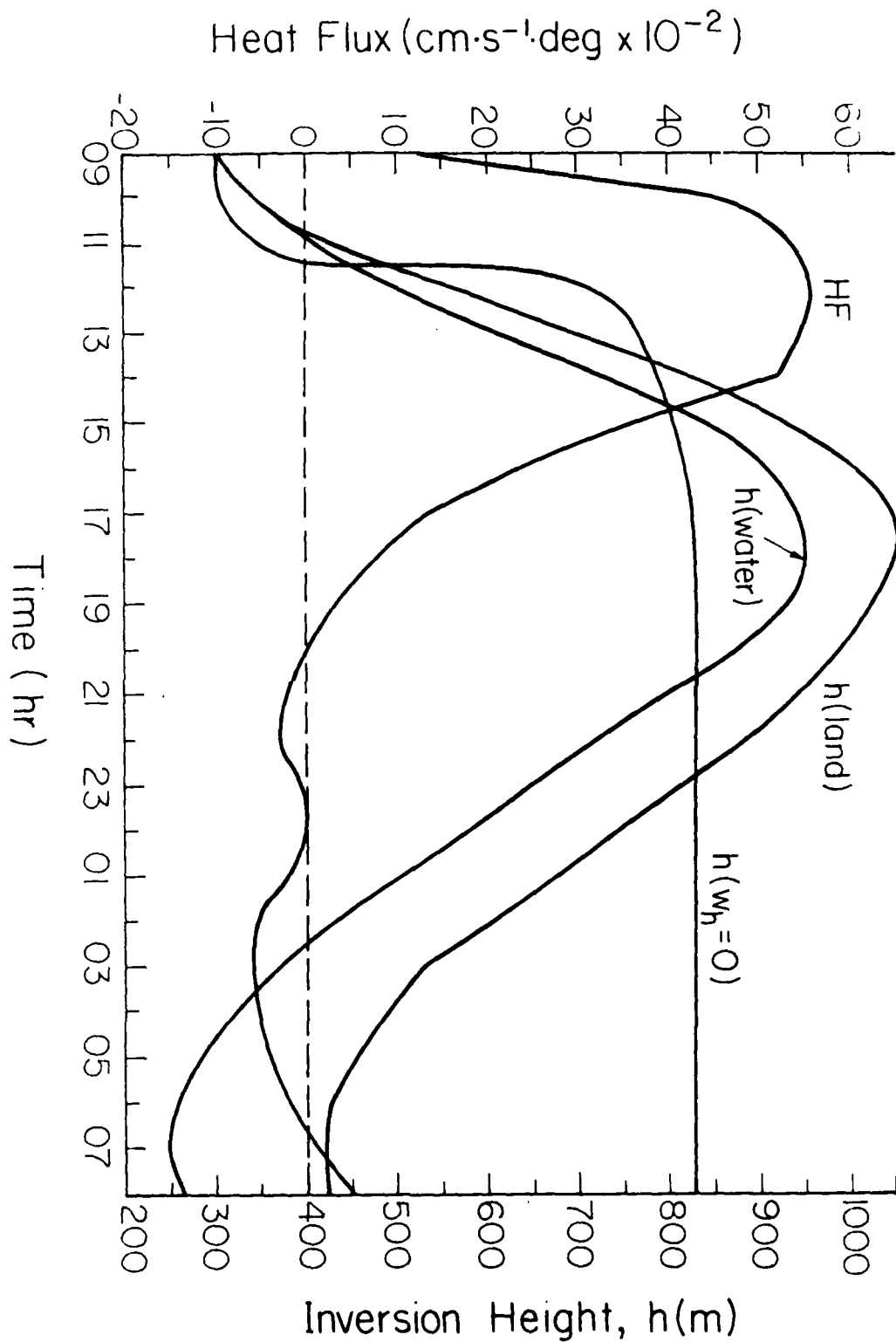


Figure 3. Inversion Heights ( $h$ ) and Surface Heat Flux (HF) vs. Time